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A note on the existence of nonoscillatory solutions of neutral differential equations

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1. Introduction and the statement of results

In this note we consider the neutral differential equation

$$\frac{d^n}{dt^n}[y(t) - p(t)y(t-\tau)] + f(t, y(\sigma(t))) = 0, \tag{1.1}$$

where $n \geq 1$ and the following conditions are assumed:

(a) $p \in C[a, \infty)$, $p(t) \geq 0$ for $t \geq a > 0$ and τ is a positive constant;

(b) $f \in C([a, \infty) \times R)$, and

$$|f(t, u)| \leq F(t, |u|), \qquad (t, u) \in [a, \infty) \times R,$$

for some continuous function $F(t, u)$ on $[a, \infty) \times [0, \infty)$ which is nondecreasing in u for each fixed $t \geq a$;

(c) $\sigma \in C[a, \infty)$, $\lim_{t \to \infty} \sigma(t) = \infty$.

By a solution of (1.1) we mean a function $y \in C[T_y, \infty)$ for some $T_y \geq a$ such that $y(t) - p(t)y(t-\tau)$ is n-times continuously defferentiable on $[T_y, \infty)$ and that (1.1) is satisfied for $t \geq T_y$. A solution of (1.1) is called nonoscillatory if it is eventually positive or eventually negative.

Recently there has been a lot of study concerning the existence of nonoscillatory solutions of neutral differential equations. For the case where $p(t)$ is a constant coefficient we refer to [2, 4, 5, 9–12, 16, 18, 20, 21]. For the case where $p(t)$ is a variable coefficient, we refer to [1, 3, 6–8, 13–15, 17, 19, 22]. Most of the existence results obtained so far, however, are established by imposing restrictive conditions on the variable coefficient $p(t)$ in (1.1) such as

$$0 \leq p(t) \leq p_0 < 1 \qquad \text{for } t \geq a, \text{ where } p_0 \text{ is a constant.} \tag{1.2}$$

In this note we investigate the existence and asymptotic behavior of nonoscillatory solutions of (1.1) with the variable coefficient $p(t)$ satisfying

$$0 \leq p_0 \leq p(t) \leq p_1 \qquad \text{for } t \geq a, \text{ where } p_0 \text{ and } p_1 \text{ are constants.} \tag{1.3}$$

Our result is the following: