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The products $\beta_s\beta_{tp/p}$ in the stable homotopy of L_2-localized spheres

Dedicated to Professor Seiya Sasao on his 60th birthday

Katsumi SHIMOMURA

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1. Introduction

The β-elements in the stable homotopy groups $\pi_*(S^0)$ of spheres at the prime > 3 are introduced by H. Toda ([20]) and generalized by L. Smith ([19]) and S. Oka ([4], [5], [6]). In [3], H. Miller, D. Ravenel and S. Wilson presented that the Adams-Novikov spectral sequence is powerful to study the stable homotopy groups of spheres, and gave the way to define the generalized Greek letter elements in its E_2-term including β-elements. S. Oka [7], [8] and H. Sadofsky [11] showed that some of those β-elements are permanent cycles.

S. Oka and the author has studied about the product of these β-elements in the homotopy groups $\pi_*(S^0)$ ([9], [12], [13], [14], [15]) and show whether or not the products of the form $\beta_s\beta_{tp/j}$ are trivial except for the case where

$$j = p,\ s = rp + 1,\ p \nmid t \quad \text{and} \quad p^{n+1} \mid r + t + p^n \qquad \text{for some } n \geq 0.$$

Here β_s for $s > 0$ and $\beta_{tp/j}$ for $j, t > 1$ are the β-elements given by L. Smith and S. Oka. In the recent work [18], A. Yabe and the author have determined the homotopy groups of L_2-local spheres, where L_2 stands for the Bousfield localization functor with respect to the Johnson-Wilson spectrum $E(2)$ with the coefficient ring $Z_{(p)}[v_1, v_2, v_2^{-1}]$ (cf. [1], [10]). In this paper we show the triviality of the product of β-elements in the homotopy groups $\pi_*(L_2S^0)$ for the above exceptional case (see Theorem 3.3). Consider the map $l_*: \pi_*(S^0) \to \pi_*(L_2S^0)$ induced from the localization map $l: S^0 \to L_2S^0$. We notice that if $l_*(x) = l_*(y)$ in the homotopy groups $\pi_*(L_2S^0)$, then $x \equiv y \bmod F_5$ in $\pi_*(S^0)$, where F_i denotes the Adams-Novikov filtration.

Together with known results, we obtain

THEOREM 1.1. *Let s and t be positive integers. Then in the homotopy groups $\pi_*(L_2S^0)$, $\beta_s\beta_{tp/p} = 0$ if and only if one of the following condition holds:*

1) $p \mid st$ *or*