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The fundamental representation of the affine Lie algebra $A_{n-1}^{(1)}$ and the Feynman path integral

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ABSTRACT. We show that the fundamental representation of the affine Lie algebra $A_{n-1}^{(1)}$ is constructed by means of the Feynman path integral on the coadjoint orbits. Using the complex white noise on the coadjoint orbit and generalizing the method of our previous papers, we compute the path integral on the coadjoint orbit of the infinite dimensional Heisenberg group, which realizes a kernel function of an irreducible unitary representation. If we modify the computation of the path integral by multiplying a divergent factor, we obtain the vertex operator for the fundamental representation of $A_{n-1}^{(1)}$.

0. Introduction

Following the method given by Alekseev, Faddeev and Shatashvili [2], we tried to compute the Feynman path integrals on the coadjoint orbits of noncompact Lie groups ([7], [8], [9], [10], [14] e.t.c.). As to the Heisenberg group, we succeeded in computing the path integrals for complex polarizations as well as real polarizations. As to semisimple Lie groups, for real polarizations, we computed the path integrals for $SL(2, \boldsymbol{R})$ ([8]). This was generalized to a certain class of noncompact real semisimple Lie groups ([10]).

For complex polarizations, however, we encountered difficulty of divergence of the path integrals even for most simple Lie groups like $SL(2, \boldsymbol{R})$ ([7]). In [8], for complex polarizations we gave an idea how to regularize the path integrals for $SU(2)$ and $SU(1, 1)$ ($\simeq SL(2, \boldsymbol{R})$) and showed that the path integrals give the kernel functions of the irreducible unitary representations. In [9], we generalized this result to arbitrary connected semisimple Lie groups which contain compact Cartan subgroups and succeeded in computing the regularized Feynman path integrals which give the kernel functions of the irreducible unitary representations realized by the Borel–Weil theorem. Later it was pointed out by Dr. Hashimoto that our idea was nothing

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