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# Solvability of degenerate elliptic problems of higher order via Leray–Lions theorem

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ABSTRACT. We present existence result for nonlinear degenerate elliptic boundary value problems of higher order. The weak solution is seeked in a suitable weighted Sobolev space using Leray–Lions theorem.

## 1. Introduction

We study a general existence theorem for *degenerate* elliptic boundary value problems for equations of higher order of the form

$$\sum_{|\alpha|\le m}(-1)^{|\alpha|}D^{\alpha}A_{\alpha}(x,u,\dots,D^{m}u)=\sum_{|\alpha|\le m}(-1)^{|\alpha|}D^{\alpha}f_{\alpha}(x)\qquad\text{in }\Omega \tag{1.1}$$

on a closed subspace $V$ satisfying

$$W_0^{m,p}(v,\Omega)\subseteq V\subseteq W^{m,p}(v,\Omega)\,,$$

where $W^{m,p}(v,\Omega)$ is a certain *weighted Sobolev space*. The degeneracy is determined by a vector function $v(x)=(v_{\alpha}(x))$, $|\alpha|\le m$, with positive components $v_{\alpha}(x)$ in $\Omega$ satisfying certain integrability assumptions.

When we deal with $V\ne W_0^{m,p}(v,\Omega)$, we always assume that $\Omega$ satisfies the *cone property* (see e.g. Adams [1]). In fact the subspace $V$ is determined by the *homogeneous boundary conditions* appearing in the boundary value problem for the equation (1.1). The case of $V=W_0^{m,p}(v,\Omega)$ corresponds to the Dirichlet problem (where formally $D^{\beta}u=0$ on $\partial\Omega$ for $|\beta|\le m-1$) and $V=W^{m,p}(v,\Omega)$ corresponds to the Neumann problem (where formally $D^{\beta}u=0$ on $\partial\Omega$ for $m\le|\beta|\le 2m-1$). However, we can also deal with *nonhomogeneous boundary value problems* considering the equation

$$\sum_{|\alpha|\le m}(-1)^{|\alpha|}D^{\alpha}A_{\alpha}(x,u+u_0,\dots,D^{\alpha}(u+u_0))=\sum_{|\alpha|\le m}(-1)^{|\alpha|}D^{\alpha}f_{\alpha}(x)\qquad\text{in }\Omega$$

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