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On maximal Riemann surfaces

Dedicated to Professor F. Maeda for his 60th birthday

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ABSTRACT. We obtain two sufficient conditions for a Riemann surface to be maximal. One is the condition $\Gamma_{h0} \cap \Gamma_{h0}^* \neq \{0\}$ and the other is the existence of a function which has the special behavior in the neighborhood of the ideal boundary.

1. Introduction

Let R be a Riemann surface. If there exists a conformal mapping $\imath$ of R into a Riemann surface $\tilde{R}$, then we call $\tilde{R}$, or more precisely the pair $(\tilde{R}, \imath)$, an extension of R. According to this definition R itself is an extension of R. An extension $(\tilde{R}, \imath)$ is called a proper extension if $\tilde{R} \backslash \imath(R) \neq \emptyset$. A Riemann surface is called maximal if it has no proper extensions. An extension $\tilde{R}$ of R is called a maximal extension if $\tilde{R}$ is a maximal Riemann surface. On the maximality of Riemann surfaces many papers have been written. Bochner [3] proved that every Riemann surface has a maximal extension. We say that a Riemann surface R has a unique maximal extension if all maximal extensions of R are conformally equivalent to one another (cf. [6]). Clearly every maximal Riemann surface has a unique maximal extension. A closed subset E of a Riemann surface R is said to be an N_D-set if every compact subset of $\varphi(U \cap E)$ is an N_D-set in the complex plane for every local chart (U, φ) on R; see [10, p. 255] for an N_D-set. Renggli [7] determined the class of Riemann surfaces which have a unique maximal extension.

THEOREM A [7, Theorem 2]. *A Riemann surface R has a unique maximal extension if and only if R is conformally equivalent to some $\tilde{R} \backslash E$, where $\tilde{R}$ is a maximal Riemann surface and E is a closed N_D-set in $\tilde{R}$.*

By a neighborhood of the ideal boundary of R we mean the exterior of

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