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On the existence of tangential limits of monotone BLD functions

Dedicated to Professor Fumi-Yuki Maeda on the occasion of his sixtieth birthday

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ABSTRACT. Our aim in this paper is to deal with the existence of tangential limits for monotone functions u in the upper half space R^n_+ of R^n satisfying

$$\int_D |\mathrm{grad}\, u(x)|^p \omega(x) dx < \infty \qquad \text{for any bounded open set } D \subset R^n_+,$$

where $p > 1$ and ω is a non-negative measurable function on R^n_+. We are mainly concerned with the case when $\omega(x) = x_n^{p-n}$, $p > n-1$, and show that u has tangential limits at boundary points except those in a small set. For this purpose, we first give a fine limit result for BLD (or p-precise) functions on R^n_+, and then apply the estimate of the oscillations of monotone functions by the p-th means of partial derivatives over balls.

In case $\omega(x)$ is of the form $g(|x|)x_n^{p-n}$, we give a condition on g for u to have a tangential limit at the origin; in case $\omega(x) = g(x_n)x_n^{p-n}$, the same condition on g will assure that u has a usual boundary limit at any point of ∂R^n_+.

1 Introduction

Our aim in this paper is to study the existence of tangential boundary limits of monotone functions u in the half space $R^n_+ = \{x = (x_1, \ldots, x_n) : x_n > 0\}$, $n \geqq 2$, which satisfy

$$(1) \qquad \int_D |\nabla u(x)|^p x_n^{p-n} dx < \infty \qquad \text{for any bounded open set } D \subset R^n_+,$$

where ∇ denotes the gradient; note that u is locally p-precise in R^n_+ in the sense of Ohtsuka [16]; see also Ziemer [21]. Here a continuous function u is said to be *monotone* (in the sense of Lebesgue) on an open set $G \subset R^n$ if

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