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The K_*-local type of the smash product of real projective spaces

Dedicated to Professor Yasutoshi Nomura on his sixtieth birthday

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ABSTRACT. We have already determined the K_*-local types of the real projective spaces RP^n and the stunted real projective spaces RP^n/RP^m in [11] and [12]. The purpose of this note is to determine the K_*-local types of the smash products of these two projective spaces.

0. Introduction

Given a ring spectrum E with unit, a CW-spectrum X is said to be quasi E_*-equivalent to a CW-spectrum Y if there exists an equivalence $h: E \wedge Y \to E \wedge X$ of E-module spectra. A map $f: Z \to X$ is said to be quasi E_*-equivalent to a map $g: W \to Y$ if there exist equivalences $h: E \wedge Y \to E \wedge X$ and $k: E \wedge W \to E \wedge Z$ of E-module spectra such that the equality $(1 \wedge f)k = h(1 \wedge g): E \wedge W \to E \wedge X$ holds. In this case the cofiber $C(f)$ is quasi E_*-equivalent to the cofiber $C(g)$. In particular, a map $f: Z \to X$ is said to be E_*-trivial if it is quasi E_*-equivalent to the trivial map, thus $1 \wedge f: E \wedge Z \to E \wedge X$ is trivial. Let KO and KU be the real and complex K-spectrum, respectively, and S_K denote the K_*-localization of the sphere spectrum S. Recall that two CW-spectra X and Y have the same K_*-local type if and only if X is quasi S_{K_*}-equivalent to Y (see [3] or [6]). In [9] and [10] we determined the quasi KO_*-equivalent types of the real projective spaces RP^n and the stunted real projective spaces $RP^n_{m+1} = RP^n/RP^m$, and then in [11] and [12] we established to determine completely the K_*-local types of these projective spaces after investigating the behavior of their real Adams operations ψ^k_R. The purpose of this note is to determine the K_*-local types of the smash products of these two projective spaces, which allows us to compute implicitly their J-groups as well as their KO-groups (see [16] for the computation of their KO-groups with ψ^k_R).

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