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On random Clarkson inequalities

Dedicated to Professor Satoru Igari on his sixtieth birthday

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ABSTRACT. We prove Tonge's random Clarkson inequalities given for L_p in a generalized setting, where the *unknown* absolute constant appearing in his original inequalities is taken to be 1. As a corollary these inequalities for fairly many other Banach spaces such as $L_p(L_q)$, $W_p^k(\Omega)$ and c_p, etc. are immediately obtained.

Introduction

In connection with generalized Clarkson's inequalities (Kato [5]; see also the recent book [10]), a high-dimensional version of Clarkson-Boas-Koskela-type inequalities (cf. [3], [2], [8]), Tonge [12] presented random Clarkson inequalities for L_p.

In this article we prove the random Clarkson inequalities for a Banach space satisfying (p, p')-Clarkson's inequality ($1 \leqq p \leqq 2$); further the *unknown* absolute constant included in Tonge's original inequalities is replaced here by one. This enables us to obtain these inequalities for fairly many other Banach spaces, e.g., L_q-valued L_p-spaces $L_p(L_q)$, Sobolev spaces $W_p^k(\Omega)$ and the spaces c_p of p-Schatten class operators, etc (cf. [2], [4], [6], [9]; see also [10]).

In what follows, let p', q', ... denote the conjugate exponents of p, q, ... Let us first recall the generalized Clarkson inequalities: Let $A_n = (\varepsilon_{ij})$ be the Littlewood matrices, that is,

$$A_1 = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \qquad A_{n+1} = \begin{pmatrix} A_n & A_n \\ A_n & -A_n \end{pmatrix} \qquad (n = 1, 2, \ldots).$$

GENERALIZED CLARKSON'S INEQUALITIES (Kato [5], Theorem 1; cf. [12], [11], [10]). Let $1 < p < \infty$ and $1 \leqq r, s \leqq \infty$. Then, for an arbitrary positive

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