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Existence of Dirichlet infinite harmonic measures on the Euclidean unit ball

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ABSTRACT. It is shown that there exist p-Dirichlet infinite p-harmonic measures on the unit ball in the Euclidean space of dimension $n \geqq 2$ even if $1 < p < 2$. The same is also proved to be true if the p-harmonicity is generalized to the so-called $\mathscr{A}$-harmonicity of exponent p.

1. Introduction

The purpose of this paper is to give an affirmative answer to a problem originally posed by Ohtsuka [11, Chap. VIII] whether there exists a p-harmonic measure on the unit ball in the n-dimensional Euclidean space $\boldsymbol{R}^n$ $(n \geqq 2)$ with an infinite p-Dirichlet integral for the exponent $1 < p < 2$. Actually Ohtsuka raised the question in terms of extremal distances in an equivalent to but superfacially different from the above formulation. However in this paper we will confine ourselves to the frame of harmonic measures as mentioned above.

We will discuss the problem in a broader potential theoretic setting of $\mathscr{A}$-harmonicity than that of mere p-harmonicity. Following the monograph [2] of Heinonen, Kilpeläinen and Martio (see also Maz'ya [6]), we say that $\mathscr{A}$ is a strictly monotone elliptic operator on the Euclidean space $\boldsymbol{R}^n$ of dimension $n \geqq 2$ with exponent $1 < p \leqq n$ if $\mathscr{A}$ is a mapping of $\boldsymbol{R}^n \times \boldsymbol{R}^n$ to $\boldsymbol{R}^n$ satisfying the following five conditions (2)–(6) for some constants $0 < \alpha \leqq \beta < \infty$:

(2) the function $h \mapsto \mathscr{A}(x,h)$ is continuous for almost every fixed $x \in \boldsymbol{R}^n$, and the function $x \mapsto \mathscr{A}(x,h)$ is measurable for all fixed $h \in \boldsymbol{R}^n$;

for almost every $x \in \boldsymbol{R}^n$ and for all $h \in \boldsymbol{R}^n$

$$\mathscr{A}(x,h) \cdot h \geqq \alpha |h|^p, \tag{3}$$

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