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Stable hypersurfaces with constant mean curvature in R^n

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ABSTRACT. This paper gives a classification of complete nonnegative Ricci curvature stable hypersurfaces with constant mean curvature in a Riemannian manifold. This theorem partially proves Do Carmo's conjecture that a complete noncompact stable hypersurface in R^{n+1} with constant mean curvature is minimal.[1]

1. Introduction

Every manifold in this paper will be orietable. Ever since Barbosa and do Carmo [1, 2] generlized the definition of stable minimal hyprersurfaces to stable hypersurfaces with constant mean curvature, much research has been done to classify these kinds of hypersurfaces. Compact hypersurfaces with constant mean curvature in a Riemannian manifold, if they are stable, were classified by Barbosa-do Carmo [2] as geodesic spheres. For noncompact stable surfaces in a 3-dimensional manifold, da Silveira [6] gave a complete classification. For the higher dimensional case, Do Carmo [3] made the following conjecture based on Chern's paper [5] on classification of graphies in R^n and da Silveira theorem:

CONJECTURE 1.1. *A complete noncompact stable hypersurface* $X: M^n \hookrightarrow R^{n+1}$ *with constant mean curvature is minimal.*

By the following theorem we get an affirmative answer to the conjecture when M has nonnegative Ricci curvature:

THEOREM 1.1. *Let* $X: M^n \hookrightarrow N^{n+1}(c)$ *be a complete noncompact stable hypersurface with constant mean curvature, and* $N^{n+1}(c)$ *be an* $(n+1)$-*dimensional Riemannian manifold whose sectional curvature is* c,

(1) *If* $c = 0$ *and* $\mathrm{Ricci}(M) \geq 0$, *then* M *must be a plane.*
(2) *If* $c = 1$, *it is impossible that* $\mathrm{Ricci}(M) \geq 0$.

REMARK: Any convex hypersurfaces in an Euclidean space must have

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