HIROSHIMA MATH. J.
26 (1996), 557–572

Linearized oscillations for neutral equations I: Odd order[1]

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(Received January 31, 1994)
(Revised May 29, 1995)

ABSTRACT. In this paper we prove that, under appropriate hypotheses, the nonlinear neutral delay differential equation $\frac{d^n}{dt^n}[x(t) - p(t)g(x(t-\rho))] + q(t)h(x(t-\delta)) = 0$ has the same oscillatory character as its associate linear equation for the case when n is odd. Our result can also be applied to the case when $p(t)$ itself is allowed to change its sign.

1. Introduction

Neutral delay differential equations are differential equations in which the highest order derivative of the unknown function appears both with and without delays. The theory of neutral equations is of both theoretical and practical interest. Neutral delay differential equations appear in networks containing lossless transmission lines (as in high speed computers where the lossless transmission lines are used to interconnect switching circuits), in the study of vibrating masses attached to an elastic bar and also as the Euler equations in some variational problems. See Driver [3], Hale [4] and the references cited therein.

Consider the n^{th}-order neutral delay differential equation

$$\frac{d^n}{dt^n}[x(t) - p(t)g(x(t-\tau))] + q(t)h(x(t-\delta)) = 0 \tag{1.1}$$

where n is an odd positive integer,

$$p, q \in C([t_0, \infty), \boldsymbol{R}), g, h \in C(\boldsymbol{R}, \boldsymbol{R}), \qquad \tau > 0 \quad \text{and} \quad \delta \geq 0. \tag{1.2}$$

The linearized oscillation theory for (1.1) was first studied in Ladas and Qian [8], see also Gyori and Ladas [5]. They proved

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1991 *Mathematics Subject Classification.* Primary 34K40, Secondary 34C10.

Key words and phrases. Oscillation, neutral equation, linearized equation.

[1]Research partially supported by NSERC of Canada and NNSF of China. Research was done when the second author was visiting the University of Alberta.