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Radial symmetry of positive solutions for semilinear elliptic equations in a disc

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ABSTRACT. Symmetry and monotonicity properties of positive solutions of the problems $\Delta u + f(|x|, u) = 0$ in D and $u = 0$ on ∂D are considered, where D is the unit disc in $\boldsymbol{R}^2$. We give to D the Poincaré metric and then employ the moving plane method to obtain new theorems on symmetry. We also consider singular solutions.

1. Introduction

This paper is concerned with symmetry and monotonicity properties of positive solutions of the problems

$$(1.1)\qquad \begin{cases} \Delta u + f(|x|, u) = 0 & \text{in } D, \\ u = 0 & \text{on } \partial D, \end{cases}$$

and

$$(1.2)\qquad \begin{cases} \Delta u + f(|x|, u) = 0 & \text{in } D \backslash \{0\}, \\ u = 0 & \text{on } \partial D, \\ \lim\limits_{|x| \to 0} u(x) = \infty, \end{cases}$$

where $D = \{x \in \boldsymbol{R}^2 : |x| < 1\}$.

There is much current interest in the symmetry properties of solutions of the problems (1.1) and (1.2). Assume that $f(r, u)$ is decreasing in r. Then according to Gidas-Ni-Nirenberg's theorem [5], any nonnegative solution $u \in C^2(\bar{D})$ of (1.1) is rotationally symmetric. Their proof is based upon Alexandrov's moving plane method. Among other results, in [6], Lazer and McKenna have proved the following: assume that

$$\frac{\partial f}{\partial u}(r, u) < \lambda_2 \qquad \text{for } (r, u) \in ([0, 1] \times \boldsymbol{R}),$$

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