Hiroshima Math. J.
26 (1996), 515–529

Royden compactification of integers

Janusz Wysoczański

(Received January 27, 1995)

Abstract. We present an explicit description of the Royden compactification of the discrete topological space $\boldsymbol{Z}$ of integers. It is defined to be the Gelfand space of the Royden algebra of bounded, Dirichlet finite functions. We identify it with a quotient space of the Čech-Stone compactification $\beta\boldsymbol{Z}$. The quotient map is expressed in terms of properties of subsets of $\boldsymbol{Z}$. Moreover, the quotient topology is also described in such a way.

1. Introduction

In the classification theory of Riemann surfaces, significant roles are played by different boundaries, invented by Wiener, Martin, Royden and others. The boundary defined by Royden (see [5]) is one of the most fruitful concepts in the theory. For a given Riemann surface R its Royden compactification R^* is a space which satisfies the following conditions (see also [7], Chapter III.):

(R1) R^* is a compact Hausdorff space.

(R2) R^* contains R as an open dense subspace.

(R3) Every function from the Royden algebra $BD(R)$ extends to a continuous function on R^*.

(R4) The Royden algebra $BD(R)$ separates points in R^*.

It is known that the compactification R^* of R exists and is unique up to a homeomorphism fixing R pointwise.

An analogous theory is developed in [8] and [9] for graphs instead of Riemann surfaces (or for more general structures called electrical networks), where the above is also true. The Gelfand theory of representations of commutative Banach algebras yields the existence of the Royden compactification for an infinite connected graph. It is identified with the Gelfand space of the Royden algebra BD. The general theory doesn't say, however, how the compactification looks. In general, there are no known examples of the Royden compactification for graphs.

1991 *Mathematics Subject Classification*. Primary: 54D35. Secondary: 46J10, 60J50, 54D80.

Key words and phrases. Royden compactification, Royden boundary, Čech-Stone compactification, Gelfand space, ultrafilters, quotient space, integers.

This work was supported by KBN Grant No. 4019/PB/IM/93.