HIROSHIMA MATH. J.
27 (1997), 105–139

Boundary continuity of Dirichlet finite harmonic measures on compact bordered Riemannian manifolds

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(Received October 30, 1995)

ABSTRACT. Generalizing the notion of p-harmonic measures in the sense of Heins we consider $\mathscr{A}$-harmonic measures of exponent p on the interior M of a compact bordered Riemannian manifold $\overline{M} = M \cup \partial M$ with smooth border ∂M of class C^∞ for $1 < p < \infty$. It is shown that $\mathscr{A}$-harmonic measures of exponent p with finite p-Dirichlet integrals on M can always be extended to continuous functions on $\overline{M}$ which are constantly zero or one on each connected component of ∂M if and only if $2 \leqq p < \infty$. In the appendix we consider an entirely arbitrary relatively compact subregion M of any Riemannian manifold of class C^∞ and it is shown that $\mathscr{A}$-harmonic measures of finite exponent $p > \dim M$ with finite p-Dirichlet integrals on M can always be extended to continuous functions on $\overline{M} = M \cup \partial M$ which are constantly zero or one on each connected component of the relative boundary ∂M of M.

0. Introduction

Take a compact bordered Riemannian manifold $\overline{M} = M \cup \partial M$ of dimension $d \geqq 2$ of class C^∞ with smooth border ∂M of class C^∞ (cf. §1.5 below) and fix a real number $1 < p < \infty$. Consider the quasilinear elliptic partial differential equation

$$-\operatorname{div} \mathscr{A}_x(\nabla u) = 0 \tag{0.1}$$

on the interior M of $\overline{M}$, where $\mathscr{A}_x(h) \cdot h \approx |h|^p$; the precise assumptions on $\mathscr{A}$ are listed in §2.1 below. A typical example of the equation (0.1) is the so called p-Laplace equation

$$-\Delta_p u = -\operatorname{div}(|\nabla u|^{p-2}\nabla u) = 0 \tag{0.2}$$

and thus of course the usual Laplace equation $-\Delta_2 u = -\Delta u = 0$ is included in our consideration.

A continuous weak solution of (0.1) on M is referred to as an $\mathscr{A}$-harmonic

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1991 *Mathematics Subject Classification*. Primary 31B35; Secondary 31C45, 30F25.

Key words and phrases. Dirichlet integral, harmonic measure.

This work was partly supported by Grant-in-Aid for Scientific Research, No. 06640227, Japanese Ministry of Education, Science and Culture.