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On the existence of Feller semigroups with boundary conditions III

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ABSTRACT. This paper is devoted to the functional analytic approach to the problem of construction of Feller semigroups with Ventcel' (Wentzell) boundary conditions in probability theory, generalizing the previous work. In this paper we construct a Feller semigroup corresponding to such a diffusion phenomenon that a Markovian particle moves both by jumps and continuously in the state space until it "dies" at the time when it reaches the set where the particle is definitely absorbed.

0. Introduction and results

Let D be a bounded domain of Euclidean space $\boldsymbol{R}^N$ with smooth boundary ∂D, and let $C(\bar{D})$ be the space of real-valued, continuous functions on the closure $\bar{D} = D \cup \partial D$. We equip the space $C(\bar{D})$ with the topology of uniform convergence on the whole $\bar{D}$; hence it is a Banach space with the maximum norm

$$\|f\| = \max_{x \in \bar{D}} |f(x)|.$$

A strongly continuous semigroup $\{T_t\}_{t \geq 0}$ on the space $C(\bar{D})$ is called a *Feller semigroup* on $\bar{D}$ if it is non-negative and contractive on $C(\bar{D})$:

$$f \in C(\bar{D}), \qquad 0 \leq f \leq 1 \qquad \text{on } \bar{D} \Rightarrow 0 \leq T_t f \leq 1 \qquad \text{on } \bar{D}.$$

It is known (cf. [8]) that if T_t is a Feller semigroup on $\bar{D}$, then there exists a unique Markov transition function p_t on $\bar{D}$ such that

$$T_t f(x) = \int_{\bar{D}} p_t(x, dy) f(y), \qquad f \in C(\bar{D}).$$

It can be shown that the function p_t is the transition function of some strong *Markov process*; hence the value $p_t(x, E)$ expresses the transition probability that a Markovian particle starting at position x will be found in the set E at time t.

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