

A family of Yang-Mills connections on 4-dimensional pseudo-Riemannian spaces

Dedicated to Professor Kiyosato Okamoto for his 60th birthday

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ABSTRACT. A family of Yang-Mills connections on 4-dimensional pseudo-Riemannian spaces S^4 , $S^1 \times S^3$, $S^2 \times S^2$ of respective indices are constructed by a group theoretic method. The index and the nullity of their second variations are calculated.

1. Introduction—A Review of Riemannian case

In this article we construct a family of Yang-Mills connections on 4-dimensional pseudo-Riemannian spaces S^4 , $S^1 \times S^3$, $S^2 \times S^2$ equipped with the indefinite Riemannian metrics of the index $(4, 0)$, $(1, 3)$, $(2, 2)$ respectively by a unified method. And then we study the index and nullity of their second variations at the canonical connection. We are interested especially in the compactified Minkowski space $S^1 \times S^3$. On the Riemannian space S^4 our connection is the BPST-instanton of the Hopf fibering $S^7 \rightarrow S^4$ (see Atiyah [1], Chapter II and Chapter III, 2). We review this case first from a group theoretic view point.

The BPST-instanton whose instanton number equals one can be constructed on Euclidean 4-space \mathbf{R}^4 (identified with the set H of quaternions) according to the following diagram:

$$\begin{array}{ccccc}
 & & G = SL_2(H) & \xrightarrow{\kappa} & K = \\
 & & \downarrow & & \downarrow \theta_M \\
 x \mapsto \begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix} & \nearrow i & & & Sp_2 \\
 & & G/P & & \downarrow Sp_2/Sp_1 = S^7 \\
 \mathbf{R}^4 = H = \bar{N} & \xrightarrow{j \text{ open dense}} & & = & K/M = Sp_2/Sp_1 \times Sp_1 = S^4 \\
 & & & & \downarrow Sp_1
 \end{array}$$

where we use the following notation. $G = SL_2(H) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a, b, c, d \in H \right\}$

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