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Fourier coefficients of modular forms of half integral weight, periods of modular forms and the special values of zeta functions

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ABSTRACT. The purpose of this note is to investigate relations between Fourier coefficients of modular forms of half integral weight and the special values of zeta functions of modular forms of integral weight. We shall derive that Fourier coefficients of modular forms of half integral weight at every non-square positive integer is explicitly expressed by means of the special values of zeta functions associated with modular forms determined by the image of Shimura correspondence of modular forms of half integral weight.

Introduction

Let $S_{k+1/2}(4N, \chi)$ and $S_{2k}(M, \psi)$ be the space of cusp forms of Neben-type χ and ψ and of weight $k+1/2$ and $2k$ with respect to $\Gamma_0(4N)$ and $\Gamma_0(M)$, respectively. In [4], Shimura showed the existence of a correspondence $\Psi^t_{N,k,\chi}$ between the space $S_{k+1/2}\left(4N, \chi\left(\frac{-1}{*}\right)^k\right)$ and the space $S_{2k}(2N, \chi^2)$ and he proved that $\Psi^t_{N,k,\chi}$ commutes with the action of Hecke operators. More precisely, let $f(z) = \sum_{n=1}^{\infty} a(n)e[nz]$ be a cusp form of $S_{k+1/2}\left(4N, \chi\left(\frac{-1}{*}\right)^k\right)$. If $f(z)$ is an eigenfunction of all Hecke operators, then there is a modular form $F(z) = \sum_{n=1}^{\infty} A(n)e[nz]$ of $S_{2k}(2N, \chi^2)$ such that

$$a(t) \sum_{n=1}^{\infty} A(n)n^{-s} = L(s-k+1, \chi\omega_t) \sum_{n=1}^{\infty} a(tn^2)n^{-s}$$

for every square-free positive integer t, where $L(*, \chi\omega_t)$ is the Dirichlet L function with a character $\chi\omega_t$ (cf. [4]).

In [7], Shintani established a lifting from a cusp form F of integral weight $2k$ to a cusp form f of integral weight $k+1/2$ and gave an expression

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