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A uniqueness criterion for the Riemann problem*

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ABSTRACT. A new entropy criterion (maximal dissipation condition) for the quasilinear wave equation with generally nonmonotone nonlinearity is introduced and tested on self-similar solutions of the corresponding Riemann problem. It is shown that the maximally dissipating solution exists and it is uniquely determined. The relation between the maximal dissipation principle and other entropy criteria is discussed.

Introduction

Introducing hysteresis into hyperbolic equations makes the problem easier. This fact has already been recognized recently [13, 14]. The present paper is based on another point of view: we do not assume any a priori hysteretic structure in a quasilinear wave equation and we show that nevertheless, convex hysteresis appears as a natural consequence of the maximal dissipation principle.

It is well known that systems of hyperbolic conservation laws of the type

$$u_t + f(u)_x = 0, \qquad x \in R^1, \qquad t > 0, \tag{i}$$

where $u(x, t)$ is the vector $(u_1(x, t), \dots, u_n(x, t))$, $n \in \mathbf{N}$, and $f: R^n \to R^n$ is a given function, do not admit in general global regular solutions satisfying a given initial condition $u(x, 0) = u_0(x)$ even if the data are smooth, and that weak solutions may be multiple if f is nonlinear, unless additional criteria are fulfilled [17]. These criteria (usually called entropy conditions) have been tested on the well-known Riemann problem for equation (i) which consists in choosing initial data

$$u(x, 0) := \begin{cases} u_- & \text{for } x < 0, \\ u_+ & \text{for } x \geq 0, \end{cases} \tag{ii}$$

where u_-, $u_+ \in R^n$ are given constant vectors.

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