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Asymptotic expansion of the joint distribution of sample mean vector and sample covariance matrix from an elliptical population

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Abstract. We consider the joint distribution of the sample mean vector and the sample covariance matrix based on the i.i.d. sample of size n. We give a basic lemma which can be used for deriving asymptotic expansions up to terms of $O(n^{-1})$ for the joint distribution of the sample mean vector and the sample covariance matrix. Using the lemma, we derive an asymptotic expansion for an elliptical population.

1. Introduction

Let $\overline{X}$ and S be the sample mean vector and the sample covariance matrix based on the i.i.d. sample of size n from a p dimensional probability distribution with mean vector μ and covariance matrix Ω. Let

$$Z = n^{1/2}\Omega^{-1/2}(S - \Omega)\Omega^{-1/2} \quad \text{and} \quad Y = n^{1/2}\Omega^{-1/2}(\overline{X} - \mu). \tag{1.1}$$

Then the limiting distribution of Z and Y is mutually independent normal. Wakaki [7] derived an asymptotic expansion for the joint distribution of Z and Y up to the order of $n^{-1/2}$ when the underlying distribution is an elliptical distribution. Unfortunately, the result included some miscalculations. The purposes of this paper are to correct them and to extend the result to an asymptotic expansion up to the order n^{-1}.

2. A basic lemma

In this section, we do not need the assumption that the underlying distribution is elliptical. For the validity of the following formal asymptotic expansion, we assume that the underlying distribution has a density function with respect to Lebesgue measure on R^p (see theorem 2 in Bhattacharya and Ghosh [1]).

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