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Homotopy coalgebras and k-fold suspensions

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ABSTRACT. We consider a weak and ordinary FG-coalgebra structure of order m on an object in a category relative to a pair of adjoint functors F and G and present some of its properties. We then specialize to the case when $F = \Sigma^k$, the k-fold suspension functor, and $G = \Omega^k$, the k-fold loop-space functor, and obtain weak and ordinary k-fold homotopy coalgebras of order m. We prove that any $(n-1)$-connected weak k-fold homotopy coalgebra of order m and of dimension $\leq (m+2)n - (m+1)k - m$ is equivalent to a k-fold suspension for any $k \geq 1$ and $n \geq 2$. We derive some consequences of this result.

Let X be a finite CW-complex which is $(n-1)$-connected, $n \geq 2$. In 1963 Berstein and Hilton proved that if X is a co-H-space of dimension $\leq 3n-3$, then X is equivalent to a suspension [2]. In 1970 Ganea proved that if X is a homotopy-associative co-H-space of dimension $\leq 4n-5$, then X is equivalent to a suspension [4]. Thus it would appear that an upper bound on the dimension of X (which is linear in n) together with restrictions on a homotopy-associative comultiplication of X would imply that X is equivalent to a suspension. The model for such results deals with the dual concept of an H-space. In this case, Stasheff's A_m-theory of H-spaces provides the necessary restriction on a multiplication for each m. For co-H-spaces, the next step was given by Saito who proved that a certain condition on a homotopy-associative comultiplication of X together with the dimensional restriction $\dim X \leq 5n-7$ implies that X is equivalent to a suspension [6]. However, the details of the argument are formidable and it is not clear what the next step should be. For a discussion of these matters, see [1, §5].

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