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A generalized projection pursuit procedure and its significance level

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ABSTRACT. A generalized projection pursuit procedure which maximizes *nonellipticity* is proposed. Behaviors of the maxima of a generalized moment index are investigated under elliptically symmetric distributions. An approximation formula of the significant level is derived. Examples which illustrate the present concepts and results are discussed. Performances of the maxima of the proposed index under certain interesting structure are asymptotically evaluated.

0 Introduction

Exploratory projection pursuit aims to explore nonlinear structures of high-dimensional data through its projection to a low-dimensional subspace. A basic component of projection pursuit is its projection index which is a function of direction to which data is projected and is used to measure the departure from normality (see Huber (1985)). Let X be a p-dimensional random vector and α a p-dimensional unit vector. When we consider one-dimensional exploratory projection pursuit, a function $I(\alpha)$, which measures the departure of $\alpha'X$ from normality is employed as a projection index, where the prime stands for the transpose of a matrix. In the two-dimensional case, let α and β be two unit vectors satisfying $\alpha'\beta = 0$. Then the nonnormality of $(\alpha'X, \beta'X)$ is measured by $I(\alpha, \beta)$, a function of α and β. Various types of projection indices were discussed in Huber (1985), as functionals satisfying affine invariance property. Friedman (1987), Jones and Sibson (1987) and Hall (1989) proposed projection indices based on the orthogonal polynomials. Unified views of projection indices based on the orthogonal polynomials were given by Cook, Buja and Cabrera (1993) and Iwasaki (1989). Sun (1993) gave practical comparisons of Friedman index and Hall index. Friedman index and Jones and Sibson index (the monent index) have been extended to the case of two-dimensional projection pursuit, and recent work of Nason (1995) discussed the moment index for the three-dimensional case. Further, Posse (1995) proposed a new projection index for the two-dimensional case. Several other

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