HIROSHIMA MATH. J.
27 (1997), 439–448

Construction of noncanonical representations of a Brownian motion

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(Received June 20, 1996)
(Revised September 4, 1996)

ABSTRACT. Give a Brownian motion $B = \{B(t); t \in [0, 1]\}$. For any linearly independent system $\mathbf{g} = \{g_1, g_2, \dots, g_N\}$ in $L^2[0, 1]$, we construct a Brownian motion $\bar{B}_{\mathbf{g}} = \{\bar{B}_{\mathbf{g}}(t); t \in [0, 1]\}$ which is noncanonical with respect to B. In detail, the orthogonal complement of $H_t(\bar{B}_{\mathbf{g}})$ in $H_t(B)$ is the linear span of $\{\int_0^t g_1(u)dB(u), \int_0^t g_2(u)dB(u), \dots, \int_0^t g_N(u)dB(u)\}$. As a special case, Lévy's examples of noncanonical representations of a Brownian motion are included. For the construction of $\bar{B}_{\mathbf{g}}$, we use the theory of a partial isometry. A generalized Hardy inequality is derived and applied as an important lemma.

0. Introduction.

The theory of canonical representation for a Gaussian process has been presented for the first time by Lévy [9] and later developed by Hida [5] and Cramér [2]. Especially Hida has given a systematic method for the theory of multiplicity of the canonical representation. The main results on the canonical representation after their initial articles are referred to the book of Hida and Hitsuda [6]. On the other hand, Lévy [10] has given some nontrivial examples of the noncanonical representations of a Brownian motion with respect to a given Brownian motion which is used as a standard in order to emphasize the importance of the canonical representation.

The aim of the present article is to give a general method so as to obtain a noncanonical representation of a Brownian motion, which has its own interest in connection with a generalized Hardy inequality in $L^2[0, 1]$ or in $L^2[0, \infty)$.

We give here a review of Lévy [10] in connection with the present problem. Let $B = \{B(t); t \in [0, 1]\}$ be a Brownian motion. It is proved that for each $q > -1/2$ and $q \neq 0$ a Gaussian process $\bar{B}_q = \{\bar{B}_q(t); t \in [0, 1]\}$ defined by

$$(1) \qquad \bar{B}_q(t) = \int_0^t \left(\frac{2q+1}{q} \frac{u^q}{t^q} - \frac{q+1}{q} \right) dB(u)$$

1991 *Mathematics Subject Classification.* 60G15, 60G25.
Key words and phrases. noncanonical representation, Brownian motion, Hardy's inequality, partial isometry.