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On reduced Q-functions

To Kiyosato Okamoto on his sixtieth birthday, with affection and admiration

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ABSTRACT. Schur's Q-functions with reduced variables are discussed by employing a combinatorics of strict partitions. They are called reduced Q-functions. We give a description of the linear relations among reduced Q-functions.

0. Introduction

Q-functions were introduced by Schur in his study of projective representations of symmetric groups. They are symmetric functions and, if we express them in terms of the power sum symmetric functions, each coefficient essentially gives the spin character of the symmetric group. This note deals with the *r-reduced* Q-functions which are defined by putting $p_{jr} = 0$ for $j = 1, 3, 5, \ldots$ in the power sum expression. When $r = p$ is a prime number, they play a role in p-modular projective representations of symmetric groups.

In a previous work [8] we showed that r-reduced Q-functions are weight vectors of the basic representation of the affine Lie algebra $A_{2t}^{(2)}$ $(r = 2t + 1)$ and chose a proper basis for each weight space.

To be more precise, let α_i (resp. $\alpha_i^\vee$) $(0 \leq i \leq t)$ be the simple roots (resp. coroots) of the affine Lie algebra $A_{2t}^{(2)}$ and $\delta = 2\sum_{i=0}^{t-1} \alpha_i + \alpha_t$ be its fundamental imaginary root. The irreducible representation with highest weight Λ_0 is called the basic representation, where $\Lambda_0(\alpha_i^\vee) = \delta_{i0}$. The set of weights is described by

$$P = \{w\Lambda_0 - n\delta;\ w \in W, n \in \boldsymbol{N}\},$$

where W is the Weyl group. This basic representation can be realized on the polynomial ring $\boldsymbol{C}[t_j; j \geq 1, \text{ odd and } j \not\equiv 0 \pmod r]$. In this realization each r-reduced Q-function turns out to be a weight vector. We answered in

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