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Uniqueness of nodal rapidly-decaying radial solutions to a linear elliptic equation on $\mathbf{R}^n$

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ABSTRACT. We consider a linear elliptic differential equation in the whole space and show the existence and uniqueness of nodal rapidly-decaying solutions with prescribed zeros. By using the Prüfer transformation, we give a comprehensive view to the problem. We also prove the existence and the uniqueness of solutions to the equation on the unit ball and the exterior of it with various boundary conditions.

1. Introduction

In this paper we consider the existence and uniqueness of nodal rapidly-decaying radial solutions to

$$\Delta u + \xi K(|x|)u = 0 \qquad \text{in } \boldsymbol{R}^n, \tag{1.1}$$

where $n > 2$ and $\xi > 0$ is a parameter. Concerning $K(r)$, we impose

(K) $\qquad K(r) > 0 \quad \text{on } (0, \infty), \qquad K(r) \in C^1(0, \infty), \qquad rK(r) \in L^1(0, \infty).$

Since we are interested in radial solutions, we consider the ordinary differential equation

$$\begin{cases} (r^{n-1}u_r)_r + r^{n-1}\xi K(r)u = 0, & r > 0, \\ u(0) = 1, \end{cases} \tag{1.2}$$

As for (1.2), it is unnecessary to restrict n to integer values. We do not require $u_r(0; \xi) = 0$, however, we can deduce $\lim_{r \downarrow 0} r^{n-1}u_r(r; \xi) = 0$ from $rK(r) \in L^1(0, 1)$ and (1.2) can be solved with only initial value $u(0) = 1$. Note that under (K), (1.2) has a unique global solution in the class $C[0, \infty) \cap C^2(0, \infty)$ for any $\xi > 0$ (see, e.g., Ni-Yotsutani [8]) and that any solution of (1.2) has

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