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Certain maximal oscillatory singular integrals

Dashan Fan and Dachun Yang*

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Abstract. We prove the L^p-boundedness $1 < p < \infty$, for certain maximal oscillatory singular integral operators.

1. Introduction

Let $y = (y_1, \dots, y_k) \in \mathbb{R}^k$ and $\nabla = (\partial/\partial y_1, \dots, \partial/\partial y_k)$ be the gradient on $\mathbb{R}^k$. $K(y) \in C^1(\mathbb{R}^k \backslash \{0\})$ is said to be a Calderón-Zygmund kernel if there is an $A > 0$ such that

$$|K(y)| \le A|y|^{-k}; \quad |\nabla K(y)| \le A\, |y|^{-k-1}; \tag{1}$$

$$\int_{b \le |y| \le B} K(y)\, d\sigma(y) = 0 \quad \text{for } 0 < b < B < \infty. \tag{2}$$

For a multi-index $\alpha = (\alpha_1, \dots, \alpha_k) \in \mathbb{N}^k \cup \{0\}$, we write

$$|\alpha| = \alpha_1 + \alpha_2 + \cdots + \alpha_k, \quad D^\alpha = (\partial/\partial y_1)^{\alpha_1} \cdots (\partial/\partial y_k)^{\alpha_k}.$$

Let $\mathscr{P}(y) = (P_1(y), \dots, P_n(y))$ be a polynomial mapping from $\mathbb{R}^k$ to $\mathbb{R}^n$, where each P_j, $j = 1, 2, \dots, n$, is a polynomial on $\mathbb{R}^k$. We define the degree of $\mathscr{P}(y)$ by $\deg(\mathscr{P}) = \max\{\deg(P_1), \deg(P_2), \dots, \deg(P_n)\}$.

The oscillatory singular integral $T_{\mathscr{P},\lambda} f(x)$ is defined by

$$T_{\mathscr{P},\lambda} f(x) = \int_{\mathbb{R}^k} e^{i\lambda\Phi(y)} K(y) f(x - \mathscr{P}(y))\, dy \tag{3a}$$

where $\Phi \in C^\infty(\mathbb{R}^k \backslash \{0\})$ is a real-valued function, $f \in s(\mathbb{R}^n)$, $\lambda \in \mathbb{R}$ and $K(y)$ is a Calderón-Zygmund kernel on $\mathbb{R}^k$. The maximal operator of $T_{\mathscr{P},\lambda}$ is defined by

$$T^*_{\mathscr{P},\lambda} f(x) = \sup_{\varepsilon > 0} \left| \int_{|y| > \varepsilon} e^{i\lambda\Phi(y)} K(y) f(x - \mathscr{P}(y))\, dy \right|. \tag{3b}$$

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