Hiroshima Math. J.
28 (1998), 139–147

On the union of 1-convex open sets

Makoto Abe

(Received January 28, 1997)

Abstract. A complex space X is 1-convex if it satisfies the conditions that there exists a locally finite 1-convex open covering of X of order ≤ 2, that the dimension of $H^1(X, \mathcal{O}_X)$ is at most countably infinite and that X is K-separable outside a compact set.

0. Introduction

It is well-known that the union of two Stein open sets in a complex space is not necessarily Stein. For example the union of two Stein open sets $\{(z_1, z_2) \in \mathbf{C}^2 \mid |z_1| < 1, 0 < |z_2| < 1\}$ and $\{(z_1, z_2) \in \mathbf{C}^2 \mid 0 < |z_1| < 1, |z_2| < 1\}$ in $\mathbf{C}^2$ is not Stein. Tovar [22] proved that if X is a union of two relatively compact Stein open sets D_1 and D_2 in a reduced Stein space S such that $\dim H^1(X, \mathcal{O}_X) < +\infty$, then X is also a Stein open set in S (Theorem 3 of Tovar [22] or Theorem 1.1 of Cho-Shon [4]).

We prove the following theorem which is a generalization of Theorem 3 of Tovar [22]. It also gives a generalization of Proposition 3.4 of Cho-Shon [4] on the finite simple chain Stein open covering. In the proof we use the theorem of Nguyen-Nguyen [20].

Let X be a second countable (not necessarily reduced) complex space. Then X is 1-convex if it satisfies the following three conditions.

i) *There exists a locally finite 1-convex open covering of X of order ≤ 2.*
ii) *The dimension of $H^1(X, \mathcal{O}_X)$ is at most countably infinite.*
iii) *X is K-separable outside a compact set.*

We also give a 1-convex version of the theorem of Markoe [16] and Silva [21] on the union of the monotone increasing sequence of Stein open sets. The results in this paper were announced in the author's articles [1, 2].

1. Preliminaries

Throughout this paper all complex spaces are supposed to be second countable. Let X be a (not necessarily reduced) complex space. We always

---

1991 *Mathematics Subject Classification*. 32F10, 32E10, 32E05.

Key words and phrases. 1-convex space, Stein space, K-separability, Remmert reduction.