HIROSHIMA MATH. J.
28 (1998), 113–127

The homotopy groups of an L_2-localized type one finite spectrum at the prime 2

Dedicated to Professor Teiichi Kobayashi on his 60th birthday

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(Received March 26, 1996)

ABSTRACT. In this paper we determine the homotopy groups as the title indicates. This is a grip to understand the homotopy groups of $\pi_*(L_2S^0)$, as well as the category of L_2-local CW-spectra at the prime 2. For example, the result indicates that an analogue of the Hopkins-Gross theorem on duality would require the condition $2 \cdot 1_X = 0$ if it holds at the prime 2.

1. Introduction

For each prime number p, let $K(n)_*$ denote the n-th Morava K-theory with coefficient ring $K(n)_* = F_p[v_n, v_n^{-1}]$ for $n > 0$ and $K(0)_* = \boldsymbol{Q}$. Here v_n has dimension $2p^n - 2$ and corresponds to the generators v_n of the coefficient ring $BP_* = \boldsymbol{Z}_{(p)}[v_1, v_2, \dots]$ of the Brown-Peterson spectrum BP at the prime p. A p-local finite spectrum F has type n if $K(i)_*(F) = 0$ for $i < n$ and $K(n)_*(F) \neq 0$. Let L_n denote the Bousfield localization functor with respect to the spectrum $K(0) \vee K(1) \vee \cdots \vee K(n)$ (or equivalently to $v_n^{-1}BP$) from the category of p-local CW-spectra to itself. In this paper we compute the homotopy groups of the L_2-localization of a type 1 finite spectrum W with $BP_*(W) = BP_*/(2) \otimes \Lambda(t_1, t_1^2, t_2)$ as a $BP_*(BP)$-comodule at the prime 2. Notice that S^0 is a type 0. Since W is a type 1 finite spectrum, it is closer to S^0 than a type 2 spectrum or an infinite spectrum. By virtue of Hopkins and Ravenel's chromatic convergence theorem, we can say that the homotopy groups $\pi_*(L_nS^0)$ will play a central role to understand the category of L_n-local spectra.

Besides, the Hopkins-Gross theorem says that the L_n-localization of the Spanier-Whitehead dual of a type n finite spectrum F is equivalent to the Brown-Comenetz dual up to some kind of suspension in the category of $K(n)_*$-local spectra if $p \cdot 1_F = 0$, and if the prime is large so that the Adams-Novikov

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1991 *Mathematics Subject Classification.* 55Q10, 55Q45, 55P60.

Key words and phrases. Homotopy groups, Bousfield localization, Adams-Novikov spectral sequence, Type n finite spectra.