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Pianigiani-Yorke measures for non-Hölder continuous potentials

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ABSTRACT. We prove that each non-Hölder continuous potential has a Pianigiani-Yorke measure for a Markovian factor of a given topological Markov chain under some condition. We give a uniqueness condition of the Pianigiani-Yorke measure together with a concrete example which shows the condition is essential. Moreover we give absolutely continuous Pianigiani-Yorke measures for cookie-cutter Cantor sets generated by $\mathscr{C}^1$-maps on $[0,1]$.

1. Introduction

Pianigiani and Yorke [12] introduced a conditionally invariant probability measure for a $\mathscr{C}^2$-map on a subset of a Euclidean space. The notion of conditionally invariant measure can be set in the context of sub-Markov chains with absorbing states. The probability measure is called a *Pianigiani-Yorke measure*. Lopes and Markarian [9] pointed out that the map is not necessarily in $\mathscr{C}^2$ but in $\mathscr{C}^{1+\gamma}$ for some $\gamma > 0$. More recently Collet, Martínez and Schmitt [6] proved that each Hölder continuous potential has a Pianigiani-Yorke measure for a Markovian factor of a topologically mixing Markov chain.

In this paper, we prove that each *non-Hölder* continuous potential has a Pianigiani-Yorke measure for a Markovian factor of a topologically mixing Markov chain under a weak condition (see Theorem 3.3 (i)). Proofs in this paper are more elementary and clearer than theirs. We refer to the tools in thermodynamic formalism introduced by Bowen [2], [4], Ruelle [13], [14], Keane [7] and Walters [16]. Especially we use g-measure to prove the convergence property (3.11) in Theorem 3.1. We show the uniqueness of the Pianigiani-Yorke measure under a certain condition (see Theorem 3.3 (iii)). We can see that the condition is essential by virtue of Example 2.

We can also construct a Pianigiani-Yorke measure for a Markovian factor which is not necessarily mixing (see Theorem 5.1). Applying Theorem 5.1 to a cookie-cutter map, we give its Pianigiani-Yorke measure, which is absolutely continuous with respect to the Lebesgue measure on $[0,1]$. Since potentials in

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