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Bipotential elliptic differential operators

Victor Anandam

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Abstract. The classification of the second order elliptic differential operators with locally Lipschitz coefficients in a domain in $\boldsymbol{R}^n$ is considered. Using potential-theoretic techniques, it is modelled after the biharmonic classification of Riemannian manifolds.

1. Introduction

Let Ω be a domain in $\boldsymbol{R}^n$, $n \geq 2$, and $\mathscr{L}(\Omega)$ the family of all second order elliptic differential operators with locally Lipschitz coefficients in Ω. In this article, we put the elements in $\mathscr{L}(\Omega)$ into different classes depending on the existence of certain special solutions of the operators.

The classification is modelled after (and more general than) that of the Riemannian manifolds $\mathscr{R}$ based on the existence of Green functions, biharmonic functions, biharmonic Green functions etc.

The similarity between these two classifications of $\mathscr{L}(\Omega)$ and $\mathscr{R}$ arises from the fact that the C^2-solutions of $Lu = 0$ for any $L \in \mathscr{L}(\Omega)$ and the harmonic functions defined by $\Delta u = 0$ in $R \in \mathscr{R}$ (where Δ is the Laplace-Beltrami operator on R) both satisfy locally the basic assumptions in the axiomatic potential theory of M. Brelot [4].

2. Preliminaries

Let $\mathscr{L}(\Omega)$ denote the family of all second order elliptic differential operators with locally Lipschitz coefficients defined on a domain Ω in $\boldsymbol{R}^n, n \geq 2$. Assume that the last coefficient is 0 in each $L \in \mathscr{L}(\Omega)$. Precisely, $Lu(x) = \sum_{i,j} a_{ij}(x) \dfrac{\partial^2 u(x)}{\partial x_i \partial x_j} + \sum_i b_i(x) \dfrac{\partial u(x)}{\partial x_i}$, where the a_{ij}'s are in $C^{2,\lambda}$ and the b_i's are in $C^{1,\lambda}$; $a_{ij} = a_{ji}$; and the quadratic form $\sum_{i,j} a_{ij}\xi_i\xi_j$ is positive definite for every $x \in \Omega$.

Then the C^2-functions u in an open set $w \subset \Omega$ for which $Lu = 0$ are called the L-harmonic functions in w. Such solutions satisfy the basic assumptions of

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