HIROSHIMA MATH. J.
28 (1998), 309–319

Local attractor for *n*-D Navier-Stokes system

Jan W. CHOLEWA and Tomasz DLOTKO*

(Received February 15, 1997)

ABSTRACT. The n-D Navier-Stokes system ($n \geq 3$) is studied as an abstract equation with sectorial operator in a relevant Banach space X_r consisting of divergence-free functions. Existence of the local semiflow $\{T(t)\}$ on a 'sufficiently smooth' fractional power space X_r^α is then known in advance. This makes it possible to consider a subset $V \subset X_r^\alpha$ for which an *a priori* estimate asymptotically independent of initial data for originated in V solutions may be derived. The task of the present paper is to apply authors' previous result [4] to the Navier-Stokes system proving existence of a global attractor $\mathscr{A}_{\alpha,r}$ for the semigroup $\{T(t)\}$ restricted to V. Simultaneously $\mathscr{A}_{\alpha,r}$ is shown to be a local attractor in a neighborhood of zero.

1. Introduction

Since the publication in 1934 of Leray's famous paper, progress in understanding the dynamics of the Navier-Stokes system has been steady but slow. Difficulties encountered in dealing with this system became particularly intensive when 3-D flows were studied. A new trend, permitting simpler treatment of this problem, was the semigroup L^p-spaces approach appearing e.g. in [2], [8], [9], [11], [12], [17], [21]. This approach has been followed in our previous papers [4], [3], where the dynamics of semilinear parabolic equations was studied within the *dissipative systems* theory [11]. In the present paper the authors' previous result [4] is applied to the Navier-Stokes system and the existence of a global attractor $\mathscr{A}_{\alpha,r}$ for the semigroup $\{T(t)\}$ restricted to V is proved. Simultaneously $\mathscr{A}_{\alpha,r}$ is shown to be a local attractor in a neighbourhood of zero.

1.1. Overview. In the following two subsections the Navier-Stokes system, viewed as a sectorial equation in the relevant Banach space X_r, is discussed to generate local semiflow $\{T(t)\}$ on the fractional phase space X_r^α. Applying *introductory estimates* concept of [4] (Sections 2.1, 2.2) we choose suitable

1991 *Mathematics Subject Classification*. 35Q30, 35B40, 35B45.

Key words and phrases. Navier-Stokes system, a priori estimates, semigroup of global solutions, dissipativeness, global attractor.

* The paper originated during the author's visit to the University of Queensland, Brisbane, Australia as an Ethel Raybould Visiting Fellow.