HIROSHIMA MATH. J.
28 (1998), 207–212

Classification of closed oriented 4-manifolds modulo connected sum with simply connected manifolds

Ichiji KURAZONO and Takao MATUMOTO

(Received December 5, 1996)
(Revised March 27, 1997)

ABSTRACT. The fundamental group is not changed by taking connected sum with simply connected manifolds. The closed oriented 4-dimensional manifolds with finite presentable fundamental group π are classified modulo this operation by the quotient $H_4(B\pi; \mathbf{Z})/(\mathrm{Aut}\,\pi)_*$. The relation to Lusternik-Schnirelmann π_1-category and some stable decomposition theorems are also discussed.

1. Introduction

Let M be a closed oriented 4-manifold. Then, we have a map $f : M \to B\pi = K(\pi, 1)$ to the Eilenberg-MacLane complex given by subsequently attaching cells of dimension greater than two, where π denotes $\pi_1(M)$. The map is unique up to homotopy by the obstruction theory if we fix the induced isomorphism on the fundamental group. The map determines the oriented cobordism class in $\Omega_4(B\pi)$. On the other hand any element of $\Omega_4(B\pi)$ gives a closed oriented 4-manifold N with $\pi_1(N) = \pi$ and a map $g : N \to B\pi$ by Lemma 5. The manifolds M and N will be shown to be stably equivalent in the sense that we have closed simply connected manifolds M_0 and N_0 satisfying $M \# M_0$ and $N \# N_0$ are orientation preserving diffeomorphic to each other.

Because any topological 4-manifold is smoothable possibly after taking connected sum with some copies of $S^2 \times S^2$ and a closed simply connected manifold with quadratic form E_8 [3], it is natural to restrict ourselves to smooth manifolds in this stable equivalence.

Since stably equivalent manifolds have the isomorphic fundamental group, it suffices to prove the following theorem for the classification.

THEOREM 1. *Let π be a finite presentable group. The stable equivalence classes of connected closed oriented 4-dimensional manifolds with fundamental*

Key words. 4-manifold, stable decomposition, Lusternik-Schnirelmann category
AMS classification. Geometric structures on low-dimension manifolds 57M50
Lusternik-Schnirelmann category of a space 55M30
Topology of E^4, 4-manifolds 57N13