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Pointwise Fourier inversion with Cesàro means

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ABSTRACT. Conditions for pointwise Fourier inversion using spherical Cesàro means of a given degree are established in euclidean and hyperbolic spaces.

1. Introduction

To solve the Fourier inversion problem, that is, to reconstruct an integrable function f on $\mathbf{R}^n$ from its Fourier transform $\mathscr{F}f$ one has in general to use summation methods. For example it is known that the kth Cesàro means $\int_{\|t\|\le N}(1-\|t\|/N)^k\mathscr{F}f(t)e^{2\pi i(x|t)}dt$ converge, when N tends to infinity, to $f(x)$ at every Lebesgue point x of f if $k>(n-1)/2$.

This is in general no more the case if $k\le(n-1)/2$. For example, if f is the indicator function of the unit ball in $\mathbf{R}^3$ and $k=0$, there is convergence everywhere except at $x=0$, which is a Lebesgue point. In this work we determine for a large class of functions, including the above indicator, the least value of k implying convergence at a given point.

We do this not only on $\mathbf{R}^n$ but also on the real hyperbolic space $\mathbf{H}^n$. Our results: the more differentiable the spherical mean of the function, the smaller the degree k insuring convergence, are natural and show a complete parallelism between both spaces. We emphasize that still little is known about summability for Fourier transforms on $\mathbf{H}^n$ (see [5] and its bibliography). Forming the basis of our reasonings are those of [7], specified and corrected (see the remark at the end of §6).

2. Cesàro summability: definition and elementary properties

DEFINITION 1. Let $b\in L^1_{loc}(\mathbf{R}_+)$, $k\ge0$ and $B\in\mathbf{C}$. We say that b is *(C,k)-summable to B* if $\lim_{x\to+\infty}\int_0^x(1-(t/x))^k b(t)\,dt=B$ and we write

$$\int_0^{+\infty}b(t)\,dt=B\quad(C,k).$$

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