On transversely flat conformal foliations with good measures II

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ABSTRACT. Transversely flat conformal foliations with good transverse invariant measures are Riemannian in the usual sense, namely, in the C^{∞} sense.

Introduction

In the previous paper [1] we have shown that transversely flat conformal foliations with good measures are transversely Riemannian in the $C^{1+\text{Lip}}$ sense, that is, we can find a holonomy invariant transverse Riemannian metric of class $C^{1+\text{Lip}}$. Recently, we found that this is still true even if we replace $C^{1+\text{Lip}}$ with C^{∞} . Namely, we have the following.

THEOREM A. Let (M, \mathcal{F}) be a transversely flat conformal foliation of a closed manifold M. Assume that there is a good measure on M. Then there is a transverse invariant Riemannian metric of (M, \mathcal{F}) which is of class C^{∞} , namely, (M, \mathcal{F}) is Riemannian in the usual sense.

Thus the theory for Riemannian foliations, which can be found in Molino [3] for instance, applies for such foliations. The proof of Theorem A can be done if we simply replace the metric in the previous paper [1] with one constructed in Ferrand [2]. The paper [2] is informed by H. Izeki, and the author would like to express his gratitude to him.

1. Proof of Theorem A

We recall the definitions, notations and some facts appeared in [1]. First of all, we recall the notion of good measures.

DEFINITION 1.1. A transverse invariant measure μ of (M, \mathscr{F}) is said to be good if μ has the following properties:

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