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Oscillation criteria for half-linear second order differential equations

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ABSTRACT. We study oscillatory properties of half-linear second order differential equation

$$(|y'|^{p-2}y')' + c(x)|y|^{p-2}y = 0, \quad p > 1,$$

viewed as a perturbation of the generalized Euler equation

$$(|y'|^{p-2}y')' + \frac{\gamma_0}{x^p}|y|^{p-2}y = 0,$$

where $\gamma_0 = \left(\frac{p-1}{p}\right)^p$.

1. Introduction

In this paper we deal with oscillatory properties of the half-linear second order differential equation

$$[\Phi(y')]' + c(x)\Phi(y) = 0, \tag{1.1}$$

where $c : [0, \infty) \to \mathbb{R}$ is a continuous function and $\Phi(s) := |s|^{p-1} \operatorname{sgn} s = |s|^{p-2}s$ with $p > 1$. It is known, see. e.g. [4, 10], that basic oscillatory properties of (1.1) are essentially the same as those of the linear differential equation

$$y'' + c(x)y = 0 \tag{1.2}$$

which is a special case $p = 2$ of (1.1). In particular, if x_1, x_2 are consecutive zeros of a nontrivial solution y of (1.1) then any other solution which is not proportional to y has exactly one zero in (x_1, x_2). Consequently, all solutions of (1.1) are either oscillatory or nonoscillatory.

In the last years, several papers appeared showing that oscillation criteria of Hartman, Wintner, Kamenev, Philos and others for (1.2) may be extended to

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