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# Lowest dimensions for immersions of orientable manifolds up to unoriented cobordism

*Dedicated to Professor Mamoru Mimura on his 60th birthday*

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Abstract. We determine the lowest dimension of the Euclidean space in which all $n$-dimensional orientable manifolds are immersible up to unoriented cobordism. Our study is an orientable version of the work investigated by R. L. Brown.

## 1. Introduction

The purpose of this paper is to give a complete answer to the immersion problem of orientable manifolds up to unoriented cobordism. Let $\alpha(n)$ be the number of 1 in the dyadic expansion of an integer $n$, and $v(n)$ the integer determined by $n = 2^{v(n)}(2m+1)$. We set $\beta(n) = 2n - \alpha(n) - \min\{\alpha(n), v(n)\}$. In [10; Theorem A], we studied immersions of orientable manifolds in the Euclidean space $\mathbf{R}^f$ up to unoriented cobordism, and gave a partial answer: (a) any closed orientable manifold $M^n$ for $n \geq 4$ is unoriented cobordant to a manifold which immerses in $\mathbf{R}^{\beta(n)}$; (b) if $\alpha(n) \leq v(n)$ and $n \geq 4$, then there exists an $n$-dimensional closed orientable manifold satisfying that any manifold unoriented cobordant to it does not immerse in $\mathbf{R}^{\beta(n)-1}$.

We always assume that a manifold is closed $C^\infty$ differentiable, and by *cobordant* we mean unoriented cobordant between manifolds. Then, our main results are stated as follows:

Theorem A. *Assume that $\alpha(n) > v(n)$ and $n \geq 4$. Then, $\beta(n) = 2n - \alpha(n) - v(n)$, and any orientable manifold $M^n$ is cobordant to a manifold which immerses, respectively, in $\mathbf{R}^{\beta(n)-1}$ or $\mathbf{R}^{\beta(n)-2}$ if the following* (1) *or* (2) *holds*:

(1) $\alpha(n) + v(n)$ *is odd, or*

(2) $\alpha(n) + v(n)$ *is even and* $n \equiv 0$ *or* $3 \pmod 4$.

Theorem B. *Assume that $\alpha(n) > v(n)$ and $n \geq 4$ with $n \neq 6, 7$. Then, there exists an $n$-dimensional orientable manifold satisfying that any manifold cobordant to it does not immerse, respectively, in $\mathbf{R}^{\beta(n)-2}$, $\mathbf{R}^{\beta(n)-3}$ or $\mathbf{R}^{\beta(n)-1}$ if the following* (1), (2) *or* (3) *holds*:

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