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On the vector field problem for product manifolds

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ABSTRACT. Let *Span*(M) be the largest number of linearly independent tangent vector fields on the manifold M. In this paper we establish a criterion giving an upper bound for *Span*(M) when M is a product of stably complex manifolds. We obtain explicit upper bounds and exact values of *Span*(M) in some special cases, such as products of lens spaces, products of quaternionic spherical space forms and products of Dold manifolds.

1. Introduction

Let M be a smooth, closed (i.e. compact and without boundary), connected manifold, we denote *Span*(M), the largest number of everywhere linearly independent tangent vector fields on M. Finding *Span*(M) is a classical problem in differential topology. This problem was solved when M is a sphere by A. Hurwitz, J. Radon and J. F. Adams (see [11], [20] and [1]). For spherical space forms, J. C. Becker has calculated *Span*(M) in [6]. For more details about the present state of the question, the reader may consult the survey paper of J. Korbaš and P. Zvengrowski [17].

In this paper we shall study *Span*(M) for M being a product of two stably complex manifolds M_1 and M_2. In other words, we suppose that the stable class of the tangent bundle τ_{M_i} of M_i carries a complex structure for $i = 1, 2$. We shall prove the following criterion for *Span*(M) in the framework of complex K-theory.

THEOREM 1.1. *Let M_i be a smooth, closed and connected stably complex m_i-manifold and let $y_i \in \widetilde{KU}(M_i)$ be the stable class represented by the tangent bundle τ_{M_i}, $(i = 1, 2)$. If Span$(M_1 \times M_2) = m_1 + m_2 - k$, then the following relation is valid in $KU^0(M_1) \otimes KU^0(M_2)$,*

$$2^{n-1}\gamma_{1/2}(y_1) \otimes \gamma_{1/2}(y_2) \equiv 0 \ (\mathrm{mod}\ 2^{n-j-1}),$$

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