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A mixed finite element method to solve the Stokes problem in the stream function and vorticity formulation

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ABSTRACT. The aim of this work is to present a new mixed finite element method to solve the two-dimensional Stokes problem in terms of the stream function and the vorticity. The main feature of the method is to overcome the difficulty associated with the lack of boundary conditions for the vorticity on a no-slip boundary, by means of an incorporated uncoupling technique of both variables.

Introduction

Let us begin by recalling the problem to solve: Given a field of volumetric forces f and denoting by ν the kinematic viscosity of the fluid occupying a bounded simply connected domain Ω of R^2 with boundary Γ, we wish to find the stream function ψ and the vorticity ω such that:

$$(0.1)\qquad \begin{cases} -\nu\Delta\omega = \text{curl } f & \text{in } \Omega \\ -\Delta\psi = \omega & \text{in } \Omega. \end{cases}$$

We consider the case where the following boundary conditions apply:

$$(0.2)\qquad \begin{cases} \psi = g_0 & \text{on } \Gamma \\ \frac{\partial\psi}{\partial n} = g_1 & \text{on } \Gamma, \end{cases}$$

where g_0 and g_1 are functions determined from the given velocity on Γ (Cf. [8]).

Long since some advantages are recognized in using the stream function-vorticity formulation to solve computationally the two-dimensional incompressible Navier-Stokes equations. Among those let us mention the fact that the former leads to computed flow fields that implicitly satisfy the incompressibility condition; moreover the number of unknown functions is reduced from three to two and the vorticity is computed directly instead of being obtained from the velocity field.

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