HIROSHIMA MATH. J.
29 (1999), 371–434

Determinantal varieties associated to rank two vector bundles on projective spaces and splitting theorems

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(Received November 9, 1998)

ABSTRACT. Introducing determinantal varieties associated to rank two vector bundles on complex projective n-spaces $\mathbf{P}^n$ ($n \geq 4$), we obtain two main splitting theorems for those bundles. By studying irreducible components of the Hilbert scheme of $\mathbf{P}^n$ containing those determinantal varieties it is shown that a rank two bundle E on $\mathbf{P}^n$ ($n = 4$ or 5) splits if and only if the first cohomology of the sheaf of endomorphisms of E vanishes. In addition, another cohomological criterion for the splitting of E is also shown using specific divisors of a determinantal variety X associated to E.

0. Introduction

(0.1) As for the splitting problems for rank two vector bundles on complex projective n-space $\mathbf{P}^n$, R. Hartshorne (cf. [14], [15], [33]) posed the following famous conjectures:

S_n: Every rank two algebraic vector bundle on $\mathbf{P}^n$ ($n \geq 7$) splits into line bundles or the conjecture which is equivalent to S_n:

C_n: Every smooth closed subvariety of codimension 2 in $\mathbf{P}^n$ ($n \geq 7$) is a complete intersection.

Later, H. Grauert and M. Schneider [11] tried to solve the following important problem. However there was a gap in their proof unfortunately.

GS: Every rank two unstable algebraic vector bundle on $\mathbf{P}^4$ is a direct sum of line bundles.

Though many mathematicians have tried to solve the conjectures S_n, C_n and the problem *GS*, e.g., W. Barth and A. Van de Ven [2], Z. Ran [31], Th. Peternell, J. Le-Potier and M. Schneider [29], ..., for almost thirty years, we have not had obtained any complete answers yet.

If we could solve the above conjectures or problem affirmatively, then it should bring us many useful results. For example:

1) It is well known that every algebraic vector bundle on the projective

Mathematics Subject Classification. 14C05, 14C20, 14J29, 14J30, 14J60.

Key words and phrases. Vector bundles, Algebraic varieties of General type, Hilbert schemes. This work was partly supported by Grant-in-Aid for Scientific Research, No. 06640054, No. 08454007, Japanese Ministry of Education, Science and Culture.