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Induction of nilpotent orbits for real reductive groups and associated varieties of standard representations

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ABSTRACT. In [LS], Lusztig and Spaltenstein introduced the notion of induction of nilpotent orbits for complex reductive groups. It is known that the induction of representations and that of nilpotent orbits are compatible with respect to the operation taking associated variety for complex reductive groups (cf. [BV]). In this paper, we give a definition of induction of nilpotent orbits by θ-stable paraboric subalgebras and that by real paraboric subalgebras for real reductive groups, and show that the generic K-orbits in the associated varieties of certain standard $(\mathfrak{g}, K)$-modules can be described by using these inductions.

0. Introduction

Let G be a complex connected reductive algebraic group and $\tau : G \to G$ a complex conjugation which defines a real form $G(\mathbf{R})$ of G. Let $\theta : G \to G$ be a (complexified) Cartan involution of G which commutes with τ. Write $K = \{g \in G; \theta(g) = g\}$ and $\mathfrak{g} = \mathfrak{k} + \mathfrak{s}$ the Cartan decomposition with respect to θ. For a closed subgroup H of G, we denote its Lie algebra by the corresponding small German letter $\mathfrak{h}$.

In §1, to describe the $\mathfrak{g}$-principal (i.e. regular in $\mathfrak{g}$) K-orbits in the associated varieties of certain standard $(\mathfrak{g}, K)$-modules, we give a parametrization of $\mathfrak{g}$-principal nilpotent K-orbits in $\mathfrak{s}$.

In §2, we discuss the induction of nilpotent K-orbits. For a θ-stable (resp. τ-stable) parabolic subgroup $Q = LU$ (resp. $P = MN$) with θ-stable and τ-stable Levi factor L (resp. M), we define

$$Ind^{\theta}((\mathfrak{l}, \mathfrak{q}) \uparrow \mathfrak{g}) : 2^{\mathcal{N}_{\mathfrak{l} \cap \mathfrak{s}}/L \cap K} \to 2^{\mathcal{N}_{\mathfrak{s}}/K}$$

$$(\text{resp.} \quad Ind^{\mathbf{R}}((\mathfrak{m}, \mathfrak{p}) \uparrow \mathfrak{g}) : 2^{\mathcal{N}_{\mathfrak{m} \cap \mathfrak{s}}/M \cap K} \to 2^{\mathcal{N}_{\mathfrak{s}}/K})$$

as a generalization of induction of nilpotent orbits in the complex cases, where we write $2^{\mathcal{N}_{\mathfrak{s}}/K}$ for the set of subsets of nilpotent K-orbits in $\mathfrak{s}$. We describe

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