

Stably extendible vector bundles over the quaternionic projective spaces

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Dedicated to Professor Fuichi Uchida on his 60th birthday

ABSTRACT. We show that, if a quaternionic k -dimensional vector bundle γ over the quaternionic projective space HP^n is stably extendible and its non-zero top Pontrjagin class is not zero mod 2, then γ is stably equivalent to the Whitney sum of k quaternionic line bundles provided $k \leq n$.

1. Introduction and results

Let F denote the field of the complex numbers $\mathbf{C}$, that of the real numbers $\mathbf{R}$ or the skew field of the quaternionic numbers $\mathbf{H}$, and FP^n the n -dimensional F -projective space. Two F -vector bundles V and W over a finite complex B are said to be stably equivalent if the Whitney sums $V \oplus I_a$ and $W \oplus I_b$ for some trivial F -vector bundles I_a and I_b are isomorphic as F -vector bundles.

The purpose of this paper is to study Schwarzenberger's property for vector bundles over the quaternionic projective space HP^n . Schwarzenberger ([Sc], [Hi]) has shown the fact that a k -dimensional F -vector bundle V over FP^n for $F = \mathbf{R}$ or $\mathbf{C}$ is stably equivalent to a Whitney sum of k F -line bundles if V is extendible, that is, if V is the restriction of a F -vector bundle over FP^m for any $m \geq n$. For the $\mathbf{C}$ -vector bundles over CP^n , proofs have been given by [Re] and [AM]. As for the $\mathbf{R}$ -vector bundles over RP^n , the stable splitting is also true under the assumption that V is the restriction of a vector bundle over RP^m for sufficiently large m ([Sc]). Some related results concerning vector bundles over the lens spaces are found in [KMY], [KM]. Our results mean that some additional conditions seem necessary for the quaternionic vector bundles over HP^n .

We remark that the extendible condition can be slightly weakened. A k -dimensional F -vector bundle γ over FP^n is called *stably extendible* if for

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