HIROSHIMA MATH. J.
29 (1999), 245–253

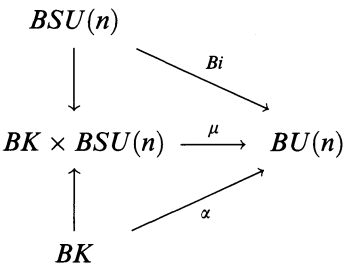
Unitary groups and pairings of classifying spaces

Kenshi ISHIGURO and Maki TOKUZAWA

(Received May 25, 1998)

ABSTRACT. We consider the maps between classifying spaces of the form $BK \times BL \to BG$. If the restriction map $BL \to BG$ is a weak epimorphism, then the restriction on BK is known to factor through the classifying spaces of the center of the compact Lie group G. Replacing the weak epimorphism $BL \to BG$ by the map $BSU(n) \to BU(n)$, analogous results are obtained. The method of our proof is, however, different from the one used for the discussion about weak epimorphisms. Namely we will use not mapping spaces but admissible maps.

The first author [9] and [10] has studied the pairing problem of classifying spaces for weak epimorphisms. In this paper we will consider the problem for a map which is not a weak epimorphism. As a test map, we take the map $BSU(n) \to BU(n)$ induced from the inclusion $i : SU(n) \to U(n)$. More precisely, for a connected compact Lie group K, we determine a subset of the homotopy set $[BK, BU(n)]$, denoted by $(Bi)^{\perp}(BK, BU(n))$, which consists of the homotopy classes of maps $\alpha : BK \to BU(n)$ such that there exists a map (called *a pairing*) $\mu : BK \times BSU(n) \to BU(n)$ satisfying $\mu|_{BK} \simeq \alpha$ and $\mu|_{BSU(n)} \simeq Bi$. We notice that $[\alpha] \in (Bi)^{\perp}(BK, BU(n))$ if and only if, for some μ, the following diagram is homotopy commutative:



Our results will indicate that the group theoretical analog also holds for some maps other than weak epimorphisms.

1991 *Mathematics Subject Classification.* 55R35, 55P15, 55P60.

Key words and phrases. Classifying spaces, admissible maps, p-compact groups, Lie groups, pairing.