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Nonlocal nonlinear systems of transport equations in weighted L^1 spaces: An operator theoretic approach

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ABSTRACT. Mathematical models of a general class for muscle contraction are studied in terms of linear semigroup theory. Two-state and four-state cross-bridge dynamics are described as nonlocal nonlinear transport systems. The initial-value problem for the nonlinear transport equation is reformulated as an abstract evolution equation in certain weighted L^1 spaces and a natural notion of mild solution to the evolution problem is introduced. The existence, blowing-up at a finite time, and uniqueness of the mild solutions are discussed under natural assumptions.

1. Introduction

This paper is concerned with nonlocal nonlinear transport systems of the form

$$\text{(NNS)} \qquad \begin{cases} \partial_t \boldsymbol{u} + z'(t)\partial_x \boldsymbol{u} = \boldsymbol{\varphi}(t, x, \boldsymbol{u}, z(t)), & (t, x) \in (0, T) \times \mathbf{R}, \\ z(t) = L\left(\displaystyle\int_{-\infty}^{+\infty} \boldsymbol{w}(y) \cdot \boldsymbol{u}(t, y) dy\right), & t \in [0, T]. \end{cases}$$

Here $\boldsymbol{u} : [0, T] \times \mathbf{R} \to \mathbf{R}^N$ is an unknown function, $[0, T]$ is a given time interval, N is a given positive integer, ∂_t and ∂_x stand for the partial differential operators with respect to the time and space variables, respectively, z' means the time derivative of z, and $\boldsymbol{w}(y) \cdot \boldsymbol{u}(t, y)$ means the inner product of $\boldsymbol{w}$ and $\boldsymbol{u}$ in $\mathbf{R}^N$. Moreover, the function $\boldsymbol{\varphi} : [0, T] \times \mathbf{R} \times \mathbf{E} \times \mathbf{R} \to \mathbf{R}^N$ is supposed to be continuous in $(t, \boldsymbol{u}, z)$, where $\mathbf{E} = \{(u^1, \dots, u^N) \in \mathbf{R}^N \mid u^1, \dots, u^N \geq 0, u^1 + \cdots + u^N \leq 1\}$, $\boldsymbol{\varphi}$ need not be continuous in x, $L : (a, b) \to \mathbf{R}$ is a continuous, decreasing function, and $\boldsymbol{w} : \mathbf{R} \to \mathbf{R}^N$ is a continuous weight function whose components are all nondecreasing. The precise assumptions for the system are made later.

The coefficient $z'(t)$ of $\partial_x \boldsymbol{u}$ in (NNS) may vanish and need not have a constant sign. Hence the system (NNS) of partial differential equations may degenerate to a system of ordinary differential equations. Further, the

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