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Quantum deformations of certain prehomogeneous vector spaces III

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Abstract. We apply our previous result [14] to the classical groups, and construct quantum analogues of the coordinate algebras of certain prehomogeneous vector spaces as non-commutative algebras equipped with actions of the quantized enveloping algebras. We also give explicit descriptions of the non-commutative counterparts for the generators of the defining ideals of the closures of orbits including basic relative invariants. In particular, quantum analogues of a quadratic form and the determinant of a symmetric matrix are naturally obtained.

0. Introduction

Let L be a connected reductive algebraic group over the complex number field $\mathbf{C}$, and let $\mathfrak{l}$ be the Lie algebra of L. We denote by $U_q(\mathfrak{l})$ the quantum deformation of the enveloping algebra $U(\mathfrak{l})$ of $\mathfrak{l}$ constructed by Drinfel'd [1] and Jimbo [5]. It is a Hopf algebra over the rational function field $\mathbf{C}(q)$. By Lusztig [6] any finite dimensional $\mathfrak{l}$-module V has a quantum deformation V_q as a $U_q(\mathfrak{l})$-module. In order to investigate quantum analogues of results concerning geometric structure of V such as L-orbits, we need also a quantum deformation of the coordinate algebra $A(V)$. In this paper we shall construct a quantum deformation $A_q(V)$ of the coordinate algebra $A(V)$ for certain prehomogeneous vector spaces V, and give counterparts for the defining ideals of the closures of L-orbits on V and their canonical generator systems.

More generally, let X be an affine variety endowed with an action of L. Then $A(X)$ is a right $A(L)$-comodule whose coaction

$$\tau : A(X) \to A(X) \otimes A(L)$$

is an algebra homomorphism. Thus we obtain a locally finite left $U(\mathfrak{l})$-module structure on $A(X)$ satisfying

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