HIROSHIMA MATH. J.
30 (2000), 29–53

Modification of *AIC*-type criterion in multivariate normal linear regression with a future experiment

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(Received December 9, 1998)
(Revised March 2, 1999)

ABSTRACT. In this paper we propose a modification of *Predictive AIC* which is an extension of *AIC* to an extrapolation case. This modification reduces bias for both the cases when a candidate model contains the true model and even when it does not contain the true model. Simmulation study shows that our criterion has also a good property in the mean square error.

1. Introduction

We consider multivariate linear regression of response variables $y_1, \dots, y_p$ on a subset of k_F explanatory variables $x_1, \dots, x_{k_F}$. Suppose that there are n observations of $y' = (y_1, \dots, y_p)$ for each fixed explanatory variables $x_F' = (x_1, \dots, x_{k_F})$. Let Y be an $n \times p$ current observation matrix and X_F an $n \times k_F$ current regression matrix. The multivariate linear regression model including all explanatory variables is written as

$$Y = X_F \Theta_F + \mathscr{E}, \qquad \mathscr{E} \sim N_{n\times p}(\mathrm{O}_{n\times p}, \Sigma_F \otimes I_n),$$

where Θ_F is a $k_F \times p$ matrix of unknown parameters, $\mathscr{E}$ is an $n \times p$ error matrix and the rows of $\mathscr{E}$ are assumed to be independently distributed as a p-variate normal distribution with mean zero and covariance matrix Σ_F. We call the model *current full model* or *full model*. The multivariate linear regression has been disscussed in both theoritical and applied statistics, e.g., in a theoritical statistics (Anderson (1958), Rao (1973), Silvey (1970)) and in an applied statistics (Chatterjee and Price (1977), Draper and Smith (1966), Seber (1977)). Mainly our disscussions are based on a multivariate normal distibution. Since our regression model has an normal distributed error matrix, the probability density function of the observation matrix under the full model is given by

$$f_F^Y(Y|\Theta_F, \Sigma_F) = (2\pi)^{-np/2}|\Sigma_F|^{-n/2}\exp\{-\tfrac{1}{2}\mathrm{tr}(Y - X_F\Theta_F)'(Y - X_F\Theta_F)\Sigma_F^{-1}\}.$$

2000 *Mathematics Subject Classification.* 62H12, 62F7.

Key words and phrases. *AIC*, bias reduction, extrapolation, model selection, normal linear regression, small sample.