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Biharmonic extensions in Riemannian manifolds

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ABSTRACT. We give a characterization of the hyperbolic Riemannian manifolds R in which for any biharmonic function b outside a compact set, there exists a biharmonic function B in R such that $B - b$ is bounded outside a compact set.

1. Introduction

Let R be a hyperbolic Riemannian manifold. It is known that given a harmonic function h outside a compact set, there always exists a harmonic function H in R such that $H - h$ is bounded outside a compact set. One method of proof of this is via the principal functions [11] making use of the potentials >0 in R, a potential in R being a superharmonic function $u \geq 0$ in R such that if h is a harmonic function satisfying $0 \leq h \leq u$, then $h \equiv 0$. To solve a similar problem for the biharmonic functions in R, Chung [8] uses a variant of these principal functions. But the result is not satisfactory.

We prove in this note that a biharmonic extension in R is possible if and only if R satisfies the following condition: There exist potentials $p > 0$ and $q > 0$ in R such that $\Delta q = p$ where $\Delta = -\sum_{i,j} g^{ij} \frac{\partial}{\partial x_i} \frac{\partial}{\partial x_j}$ is the Laplace-Beltrami operator, and q is bounded outside a compact set. We remark that this condition is verified in $\mathbf{R}^n$, $n \geq 5$.

The proof of this biharmonic extension depends on a lemma giving the representation of a biharmonic function defined outside a compact set in R by means of the difference of some special potentials in R.

2. Preliminaries

Let R be an oriented Riemannian manifold of dimension $n \geq 2$ with local parameters $x = (x^1, \ldots, x^n)$ and a C^∞ metric tensor g_{ij} such that $g_{ij}x^i x^j$ is positive definite. We denote the volume element by $dx = \sqrt{\det(g_{ij})}\, dx^1 \cdots dx^n$;

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