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Exact solutions of a competition-diffusion system

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Abstract. In this paper, we consider a two-component competition-diffusion system of Lotka-Volterra type which arises in mathematical ecology. By introducing an appropriate ansatz, we look for exact travelling and standing wave solutions of this system.

1. Introduction

In mathematical ecology, it has been proposed that systems of reaction-diffusion equations can describe the interaction of biological species which move by diffusion. A frequently used model is the following Lotka-Volterra competition-diffusion system [7]:

$$
\begin{aligned}
\frac{\partial u}{\partial t} &= d_u \frac{\partial^2 u}{\partial x^2} + u(a_u - b_u u - c_u v), \\
\frac{\partial v}{\partial t} &= d_v \frac{\partial^2 v}{\partial x^2} + v(a_v - b_v u - c_v v),
\end{aligned}
\tag{1.1}
$$

where $u = u(x,t)$ and $v = v(x,t)$ represent population densities of two competing species which move by diffusion. The constants a_u and a_v are the intrinsic growth rates, b_u and c_v are the coefficients of intraspecific competition, b_v and c_u are the coefficients of interspecific competition, and d_u and d_v are the diffusion rates. We assume that all of these quantities are positive.

By a suitable transformation, we can rewrite (1.1) as

$$
\begin{aligned}
\frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2} + u(1 - u - cv), \\
\frac{\partial v}{\partial t} &= d \frac{\partial^2 v}{\partial x^2} + v(a - bu - v),
\end{aligned}
\tag{1.2}
$$

where the constants a, b, c, and d are positive. For the initial value problem of (1.2) with initial data $(u,v)(x,0) \geq 0$, the asymptotic behavior of (u,v) can be classified into the following four cases [2]:

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