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Stationary solutions to boundary problem for the heat equations

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ABSTRACT. The necessary and sufficient conditions for the existence of a stationary solutions to the boundary value problem for an abstract heat equation with a stationary disturbances and to the stochastic boundary value problem for such equation in the strip are given. The existence of a bounded solutions to the deterministic boundary value problem is also considered.

1. Introduction

In this paper we deal with an abstract stochastic heat equations, for which one of the independent variables represents time. It is supposed that random disturbances on the right-hand side are stationary with respect to the time variable. We are interested in solutions which are stationary with respect to the time variable of a boundary value problem in the strip. Periodic solutions for the deterministic partial differential equations are intensively studied, see, for example, well known book [15]. The problem of the existence of stationary solutions to a stochastic ordinary differential equation is also well understood, see books [8], [5] and a survey [6] for more references. During the past years it has become apparent that it is natural and more adequate in many applications to consider an input source as a random source or random disturbances. Thus investigations of stochastic partial differential equations are important. We consider the stationary solutions to some boundary value problem for a heat equation and will present some approach to obtain the existence theorem of stationary solutions. This approach is based on the results from [3] and [4]. We will demonstrate it in a simple situation relative to the random disturbances.

Let $(B, \|\cdot\|)$ be a complex separable Banach space, $\bar{0}$ the zero element in B, and $\mathscr{L}(B)$ the Banach space of bounded linear operators on B with the operator norm, denoted also by the symbol $\|\cdot\|$. For a B-valued function the continuity and differentiability means correspondingly the continuity and differentiability in the B-norm. For an operator A the sets $\sigma(A)$ and $\rho(A)$ are its spectrum and resolvent sets, respectively. Let I be the identity operator.

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