Properties of harmonic boundary in nonlinear potential theory

Dedicated to Professor Masayuki Itô for his sixtieth birthday

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(Received December 16, 1999)

ABSTRACT. We consider a quasi-linear second order elliptic differential equation on a euclidean domain, and for a compactification of the domain we define the harmonic boundary relative to the structure condition of the equation. Properties of harmonic boundary known in the classical potential theory are extended to our nonlinear case. We show that the comparison principle with respect to harmonic boundary holds for our equation, and give relations between Dirichlet-regular points and the harmonic boundary points.

Introduction

In an ideal boundary theory for Riemann surfaces, the notion of harmonic boundary has been introduced as a potential theoretically essential part of the given ideal boundary (cf. [CC]). Among others, the minimum principle with respect to harmonic boundary (cf. [CC; Satz 8.4 and Folgesatz 8.1]) and the fact that the harmonic boundary on the Royden boundary coincides with the set of all regular points with respect to the Dirichlet problem (cf. [CC; Folgesatz 9.2]) are typical results showing the importance of this notion. Such results have been also considered on Riemannian manifolds (cf. e.g., [GN]) and behavior of solutions of the equation $\Delta u - Pu = 0$ at the harmonic boundary have been studied (cf. [GKa] and [GN]). Further, these results are extended to the p-Royden boundary of a Riemannian manifold Ω, for which the minimum principle (or, rather the comparison principle) and the Dirichlet problem are considered with respect to the p-Laplacian ([T1] and [T2]) or more generally, with respect to the quasi-linear elliptic equation

$$-\operatorname{div} \mathscr{A}(x, \nabla u(x)) = 0,$$

where $\mathscr{A}(x, \xi) : \Omega \times \mathbf{R}^N \to \mathbf{R}^N$ satisfies structure conditions of p-th order with $1 < p < \infty$ (see [N]).