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Dihedral f-tilings of the sphere by triangles and well centered quadrangles

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ABSTRACT. In [1] the notion of *well centered spherical quadrangle* (WCSQ) and their properties were described. The study of dihedral f-tilings of the Riemannian sphere S^2 by spherical triangles and WCSQ was initiated in [2], where the classification by spherical triangles and equiangular spherical quadrangles was given. In [3] the classification of dihedral f-tilings by spherical triangles and spherical quadrangles of lozenge type was done. Here we complete the classification of all dihedral f-tilings of S^2 by triangles and well centered quadrangles, presenting the study of dihedral f-tilings by triangles and WCSQ with distinct pairs of congruent opposite angles and with distinct pairs of congruent opposite sides, Figure 80.

1. Introduction

Let us consider the Riemannian sphere S^2. A spherical moon L is said *well centered* if its vertices belong to the great circle $S^2 \cap \{(x, y, z) \in \mathbf{R}^3 \mid x = 0\}$ and the semi-great circle bisecting L contains the point $(1, 0, 0)$. By a *well centered spherical quadrangle* (WCSQ) we mean a spherical quadrangle which is the intersection of two well centered spherical moons with distinct vertices. In [1] it was established that any spherical quadrangle with congruent opposite internal angles is congruent to a WCSQ.

By a *dihedral f-tiling* of the sphere S^2 whose prototiles are a WCSQ Q and a spherical triangle T we mean a polygonal subdivision τ of S^2 such that each *cell* of τ is isometric either to Q or T and all vertices of τ satisfy the *angle-folding relation*.

F-tilings are intrinsically related to the theory of isometric foldings of Riemannian manifolds. See [8] for the foundations of this subject.

Isometric foldings are locally isometries which send piecewise geodesic segments into piecewise geodesic segments of the same length. These maps are

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