Generation and characterization of locally Lipschitzian semigroups associated with semilinear evolution equations

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Abstract. Nonlinear continuous perturbations of (C_0)-semigroups are treated from the point of view of the theory of semigroups of nonlinear operators. Given a (C_0)-semigroup $T(t)$ with generator A in a Banach space X, a general class of nonlinear perturbations is introduced by means of a l.s.c. functional φ. Generation and characterization of locally Lipschitzian semigroups are discussed in terms of semilinear stability condition and subtangential condition. The local Lipschitz continuity and growth condition for the semigroups are restricted by a lower semicontinuous functional φ on a Banach space X under consideration. In the case in which both φ and the domain D of the perturbing operator B are convex, it is shown that the semilinear stability condition is replaced by the standard quasidissipativity condition, and that a Hille-Yosida type theorem is obtained. Moreover, generation and characterization of locally Lipschitzian groups are investigated.

1. Introduction

As widely recognized, the theory of semilinear evolution equations plays an important role in the studies of semilinear problems arising in various fields. Of special interest are generation theorems for the corresponding nonlinear semigroups in terms of necessary and sufficient conditions. A start for the semilinear Hille-Yosida theory in a general Banach space framework was made by Oharu and Takahashi in [13], and their results were significantly generalized in [12], where fundamental properties of the semilinear generators are also investigated.

One of the features of our semilinear Hille-Yosida theory is that generation theorems are not necessarily covered by well-known results concerning nonlinear contraction semigroups. An important example in this sense is a generalized Kortweg-deVries equation, which is treated in [2] and the